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**Equivalence of Voltage Bias and  
Geometric Waveguide Design in  
Directional Couplers**

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# **Equivalence of Voltage Bias and Geometric Waveguide Design in Directional Couplers**

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## **Abstract**

Using Poincaré coordinates we show that any relative amplitude and phase between the two modes of a directional coupler can be obtained through varying a single electrode voltage and by selecting the correct length for the coupling region. Alternatively, curved waveguide sections can be employed to accomplish the same feat without applied voltage, which we demonstrate using beam propagation simulations.

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Dc biased, integrated optic, linearized directional coupler optical modulators have been extensively studied<sup>1</sup>. They suffer from the deficiencies that a large dc bias voltage with tight control is required in order to maintain approximately linear operation, and the devices are subject to dc drift<sup>2</sup> due to large fields and low-mobility ionic conductivity.

We will describe a new method to reach any arbitrary operating point for integrated optical devices that use dc bias voltages. This technique will greatly reduce the magnitude or completely eliminate the need for dc bias voltages. The technique may generally be described as the use of waveguide bends of controlled length and curvature that passively bias the modulator into another operating regime.

The propagation and coupling of optical modes in directional couplers can be represented on a Poincaré sphere<sup>3</sup>. We will use the symmetric and antisymmetric supermodes of the directional coupler. The equations governing the evolution of the complex mode amplitudes in a lossless directional coupler are<sup>4</sup>

$$\begin{aligned} a_s(z) &= e^{j\beta_{av}z} \left[ a_{s0} \cos(bz) + j \sin(bz) \left( \frac{a_{s0}\Delta\beta}{2b} + \frac{a_{a0}\chi}{b} \right) \right] \\ a_a(z) &= e^{j\beta_{av}z} \left[ a_{a0} \cos(bz) - j \sin(bz) \left( \frac{a_{a0}\Delta\beta}{2b} - \frac{a_{s0}\chi}{b} \right) \right] \end{aligned} \quad (1)$$

where  $\beta_s$  and  $\beta_a$  are the propagation constants,  $\Delta\beta = \beta_s - \beta_a$ ,  $\chi = \chi(V)$  is a function of the applied voltage,  $b = ((\Delta\beta/2)^2 + \chi^2)^{1/2}$  and  $\beta_{av} = (\beta_s + \beta_a)/2$ .  $a_s(a_a)$  is the complex amplitude of the symmetric (antisymmetric) mode. For a passive, lossless directional coupler,  $a_s(z) = a_{s0}e^{i(\beta_s z + \phi_0)}$  and  $a_a(z) = a_{a0}e^{i\beta_a z}$ , where  $a_{s0}$  and  $a_{a0}$  are real, and  $\phi_0$  is the phase difference at  $z=0$  which may be nonzero due to the taper region. We assume adiabatic

mode conversion in the taper regions for the remainder of this paper; this does not qualitatively change the results. The Poincaré coordinates<sup>3</sup> are defined by

$$\begin{aligned} S_1 &= |a_s|^2 - |a_a|^2 = P_s - P_a = \cos(\theta) \\ S_2 &= 2|a_s a_a| \cos(\phi) = \cos(\phi) \sin(\theta) \\ S_3 &= 2|a_s a_a| \sin(\phi) = \sin(\phi) \sin(\theta) \end{aligned} \quad (2)$$

where  $\phi$  is the phase difference between the modes,  $P_{s(a)}$  is the power in the symmetric (antisymmetric) mode and  $\{\theta, \phi\}$  are standard spherical angular coordinates (Fig. 1a).

For a two-guide directional coupler we may calculate the power in each of the two waveguides at the end of the coupling region ( $z > L$ ) as

$$P_1 = \frac{1}{2} |a_s + a_a|^2 = \frac{1 + S_2}{2} \quad (3a)$$

$$P_2 = \frac{1}{2} |a_s - a_a|^2 = \frac{1 - S_2}{2} \quad (3b)$$

For  $\chi = 0$ , the output state  $\{S_1, S_2, S_3\}$  as a function of  $L$  lies on the equator circle  $S_1 = 0$ . Only on this circle is  $|a_s| = |a_a|$ .  $P_1$  may vary from 0 to 1 while on the equator. However, for some devices such as linear modulators it is necessary to operate off the equator<sup>1</sup>. In order to vary  $S_1$ , we need to introduce asymmetry to couple the symmetric and antisymmetric modes. This can be done using an electrooptic or other material whose index varies with an external influence. We will now analyze how to select any point on the sphere as the output state of the directional coupler.

The spherical notation defined in Eq. (2) will be used, where  $\phi = \text{Arg}(a_s/a_a)$ ,  $0 \leq \phi \leq 2\pi$ , and  $\theta = \text{Cos}^{-1}(P_s \cdot P_a)$ ,  $0 \leq \theta \leq \pi$ . With  $a_{s0} = a_{a0} = 1/\sqrt{2}$  and  $\phi_0 = 0$ , solving for  $\theta$  using Eq. (1) results in

$$\cos(\theta) = \left( \frac{\chi \Delta\beta}{b^2} \right) \sin^2(bL). \quad (4)$$

For  $\chi=0$ ,  $\cos(\theta)=0$  as expected.  $\cos(\theta)$  can cover the entire -1 to +1 range, and  $\cos(\theta)=\pm 1$  only when  $\Delta\beta L = m\pi/\sqrt{2}$  (for  $m$  odd integer) and  $\chi L = \pm \Delta\beta L/2$ .

The phase difference  $\phi$  at  $z=L$  is obtained from Eq. (1) as

$$\phi = \tan^{-1}[\{(\Delta\beta/2 + \chi)/b\} \tan(bL)] + \tan^{-1}[\{(\Delta\beta/2 - \chi)/b\} \tan(bL)]. \quad (5)$$

For  $\chi=0$ ,  $\phi = \Delta\beta L$ , while in the limit of large  $\chi$ ,  $\phi=0$ . Any point  $\{\theta, \phi\}$  on the Poincaré sphere can be selected by varying  $\Delta\beta L$  and  $\chi$ , as we will show below.  $L$  and  $\Delta\beta$  can be chosen independently by the device designer.

Alternatively, the evolution of the Poincaré coordinates as a function of  $L$  and  $\chi$  is given by,

$$\begin{bmatrix} S_1 \\ S_2 \\ S_3 \end{bmatrix} = \begin{bmatrix} \frac{\Delta\beta}{2b} & \frac{-\chi}{b} & 0 \\ \frac{\chi}{b} & \frac{\Delta\beta}{2b} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(2bL) & -\sin(2bL) \\ 0 & \sin(2bL) & \cos(2bL) \end{bmatrix} \begin{bmatrix} \frac{\Delta\beta}{2b} & \frac{\chi}{b} & 0 \\ \frac{-\chi}{b} & \frac{\Delta\beta}{2b} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} S_{10} \\ S_{20} \\ S_{30} \end{bmatrix} \quad (6)$$

where  $S_{i0} = S_i(z=0)$ . This shows that points on the sphere are reached through successive rotations about the  $S_3$ ,  $S_1$ , and  $S_3$  axes. With  $\{S_{10}, S_{20}, S_{30}\} = \{0, 1, 0\}$ , Eq. (6) reduces to

$$\{S_1, S_2, S_3\} = \left\{ \frac{\chi \Delta \beta}{2b^2} (1 - \cos(2bL)), \left(\frac{\chi}{b}\right)^2 + \left(\frac{\Delta \beta}{2b}\right)^2 \cos(2bL), \left(\frac{\Delta \beta}{2b}\right) \sin(2bL) \right\} \quad (7)$$

We find from Eq.'s (3a) and (7) that

$$P_1 = 1 - \left(\frac{\Delta \beta}{2b}\right)^2 \sin^2(bL). \quad (8)$$

From Eq. 8 we see that  $dP_1/d\chi = 0$  when  $\chi=0$ , which is why a bias point with  $\chi \neq 0$  is necessary for a linear modulator. Let us now examine the power  $P_1$  for  $\Delta \beta L = \pi$ . When  $\chi=0$ ,  $P_1=0$  as expected.  $P_1$  reaches its maximum of one when  $bL = m\pi$ ,  $m$  integer. Thus by varying  $\chi$ ,  $P_1$  can be set to any desired value between 0 and 1. For  $2\chi/\Delta \beta = 0.8$ ,  $P_1 = 0.5$  and the slope of  $P_1$  versus  $\chi$  is approximately linear, a good operating point for a linear modulator. Then  $\{S_1, S_2, S_3\} \approx \{0.8, 0, -0.6\}$  which is indicated by the \* in Fig. 1a.

Another representation for evolution of the output state  $\{S_1, S_2, S_3\}$  is shown in Fig. 2. The starting points are all on the rim of the circle where  $\chi=0$ . As  $\chi$  grows large the trajectory is an ever tightening spiral about an axis which approaches the  $S_3$  axis.

With  $\{S_{10}, S_{20}, S_{30}\} = \{0, 1, 0\}$ , Eq. (7) may be solved for the  $\chi$  and  $L$  needed to reach the output state  $\{S_1, S_2, S_3\}$ .

$$\chi = \frac{\Delta \beta}{2} \left( \frac{S_1}{1 - S_2} \right) \quad (9)$$

$$L = \frac{1}{\Delta \beta \sqrt{1 + \left(\frac{S_1}{1 - S_2}\right)^2}} \sin^{-1} \left( S_3 \sqrt{1 + \left(\frac{S_1}{1 - S_2}\right)^2} \right) \quad (10)$$

This shows explicitly how any point on the Poincaré sphere can be reached by a suitable choice of  $\chi$  and  $L$ .

As shown above, setting the operating point of a symmetric directional coupler device off the equator requires a nonzero  $\chi$ . We will now describe a method to reach any point on the Poincaré sphere with little or no dc bias, by using a waveguide configuration which breaks the symmetry along  $x$ , thus providing coupling between the symmetric and antisymmetric modes.

The technique may generally be described as the use of waveguide bends of controlled length and curvature to passively bias the modulator. Figure 3 inset shows the waveguide design with sine bends for low loss<sup>5</sup> as defined by

$$x(z) = h \left( \frac{z}{L_b} - \frac{1}{2\pi} \sin \left( 2\pi \frac{z}{L_b} \right) \right) \quad (11)$$

where  $\{x,z\} = \{0,0\}$  is at the inflection point of the individual waveguide bends and  $L_b$  and  $h$  are defined in the Fig. 3 inset.

The beam propagation method<sup>6,7</sup> was used to simulate mode propagation, with light starting in waveguide 1. Figure 3 shows how  $S_1 = \cos(\theta)$  varies with the bias section length  $L_b$ . The graphs are flipped about the  $S_1=0$  axis when the direction of the bend is up instead of down. It may be that the limiting values for  $S_1$  of  $\pm 1$  are not achievable using this technique. For example, in the case of  $h=50 \mu\text{m}$  and  $L_b=2760 \mu\text{m}$ , we find  $S_1=-0.996$ , (or  $+0.996$  for the up bend case). For practical devices this range should be more than adequate.

Once the bias section ends, the phase difference  $\phi_t$  between the two modes may not be the desired value  $\phi_{\text{bias}}$ . A symmetric section of parallel waveguides with length

$$L_\phi = \begin{cases} (\phi_{\text{bias}} - \phi_t) / (\beta_s - \beta_a), & \phi_{\text{bias}} > \phi_t \\ (\phi_{\text{bias}} + 2\pi - \phi_t) / (\beta_s - \beta_a), & \phi_{\text{bias}} < \phi_t \end{cases} \quad (12)$$

changes  $\phi$  to the necessary value. This demonstrates that almost any bias point on the Poincaré sphere can be reached with only passive waveguide design. Any displacement in the actual bias point  $\{\theta, \phi\}$  from the design point due to fabrication variations can be corrected with a small dc bias voltage in the  $L_\phi$  section.

We have shown how a dc bias voltage and careful selection of the device length on an electrooptic directional coupler can set the operating point of the device anywhere on the Poincaré sphere. We have also shown that this can be accomplished through the use of geometrical waveguide design with zero, or a greatly reduced bias voltage. This relieves the requirement for large dc bias voltages and their attendant problems in electrooptic devices.

## References

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### Figure Captions

Figure 1. (a) Definition of Poincaré coordinates. (b) Geometry of coupler.

Figure 2. Trajectories on the Poincaré sphere as a function of  $\chi L$  (from 0-10) projected onto the  $S_2$ - $S_3$  plane. Various  $\Delta\beta L$  values are indicated. The circle represents possible states when  $\chi = 0$ . The lower axis represents the power  $P_1$  in waveguide 1.

Figure 3.  $S_1$  vs. transition length for the passively biased waveguides shown in the inset. The parameters used are:  $n = 1.5$ ,  $\Delta n = 0.005$ ,  $G = 5.1 \mu\text{m}$ , TE mode,  $\lambda = 1.3 \mu\text{m}$ ,  $h = 50 \mu\text{m}$  and  $h = 75 \mu\text{m}$ .

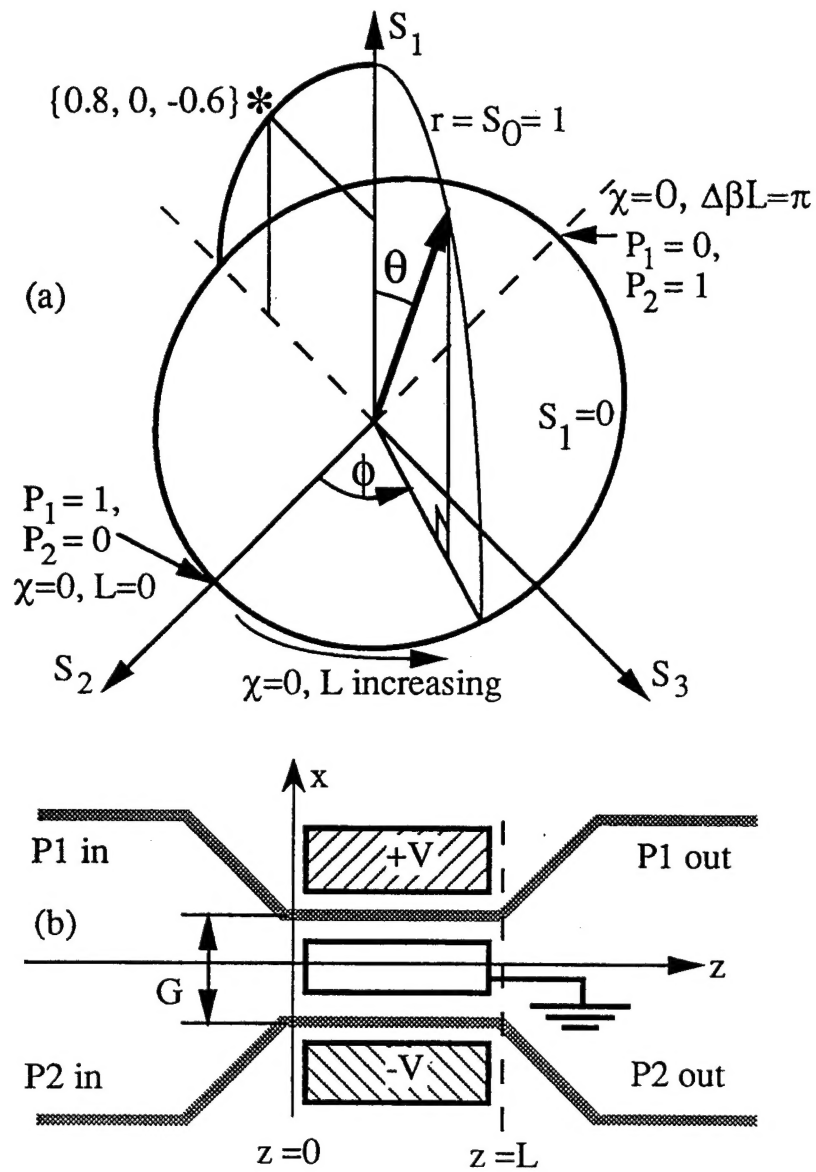


Figure 1. (a) Definition of Poincaré sphere parameters. (b) Geometry of coupler.

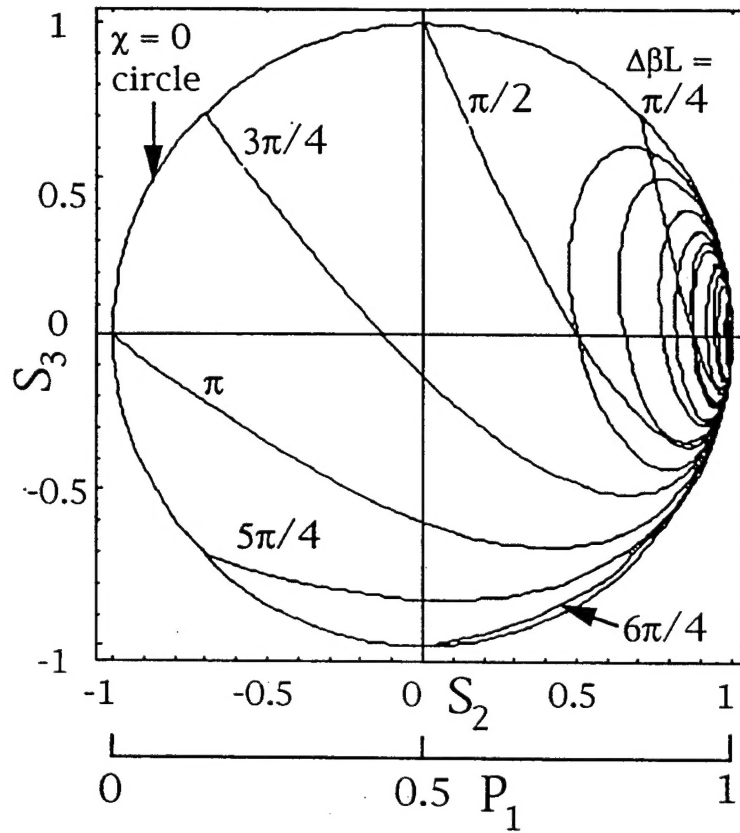


Figure 2. Projection of trajectories on Poincaré sphere to the  $S_2$ - $S_3$  plane as  $\chi$  increases from 0-10. Shown for various  $\Delta\beta L$  values as indicated. The circle is  $\chi = 0$ . Also shown is an axis for  $P_1$ .

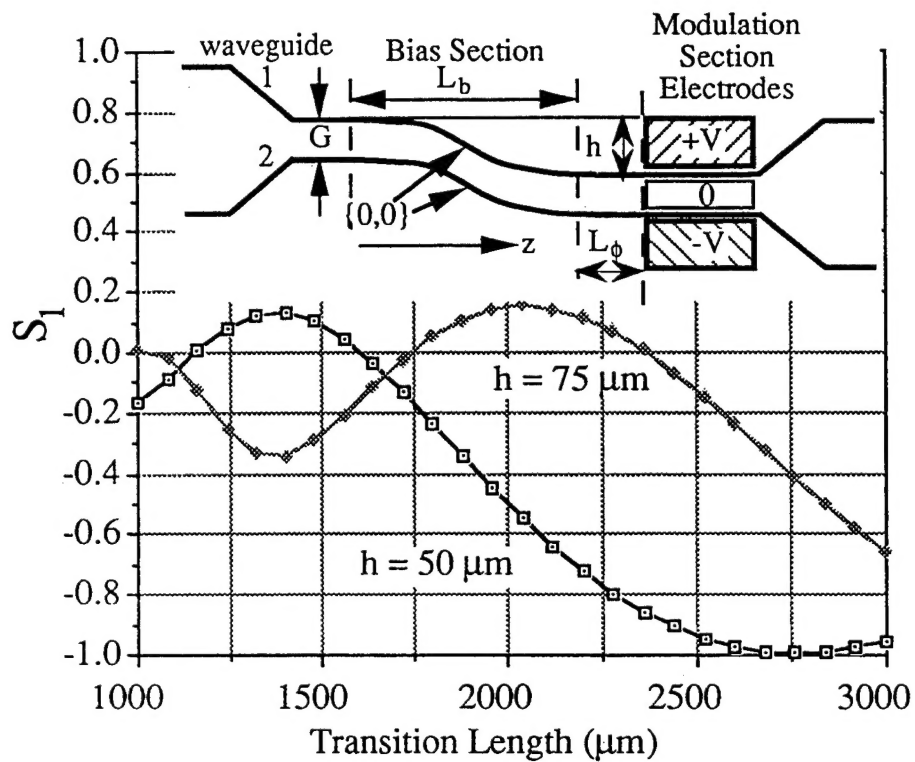


Figure 3. Plot of  $S_1$  versus transition length. The parameters used are:  $n = 1.5$ ,  $\Delta n = 0.005$ ,  $G = 5.1 \mu\text{m}$ , TE mode,  $\lambda = 1.3 \mu\text{m}$ ,  $h = 50 \mu\text{m}$  and  $h = 75 \mu\text{m}$ . Inset shows geometry.